泸州市高2022级第一次教学质量诊断性考试

数学参考答案

一．选择题

题号	1	2	3	4	5	6	7	8	9	10	11
答案	D	C	B	C	A	B	A	D	CD	ACD	BD

二．填空题

12.$(-\frac{3}{25},\frac{4}{25})$　　13.$\left[\frac{3}{2},+\infty\right)$　　14.$-\frac{10}{9}$

15.解（1）　$2\sin(2x+\frac{\pi}{3})>\sqrt{3}$

$\sin(2x+\frac{\pi}{3})>\frac{\sqrt{3}}{2}$.

$2k\pi+\frac{\pi}{3}<2x+\frac{\pi}{3}<2k\pi+\frac{2}{3}\pi$

$2k\pi<2x<2k\pi+\frac{1}{3}\pi$

$k\pi<x<k\pi+\frac{\pi}{6}$.

$\therefore f(x)>0$ 时，$x\in(k\pi,k\pi+\frac{\pi}{6}),k\in Z$

(2)$g(x)=2\sin(x+\frac{\pi}{3})-\sqrt{3}$,

$g(\frac{\pi}{3})=2\sin\frac{2}{3}\pi-\sqrt{3}=0$.

$\therefore k=g'(\frac{x}{3})=2\cos\frac{2}{3}\pi=-1$.

$\therefore y=g(x)$在$(\frac{\pi}{3},g(\frac{\pi}{3}))$处而切线方程为:.

$y=-(x-\frac{\pi}{3})$.

$x+y-\frac{\pi}{3}=0$.

16．解(1) $n=1$ 时、$2S_1=3a_1-9$

$\therefore a_1=S_1=9$.

n≥2 时

$2(S_n-S_{n-1})=3a_n-3a_{n-1}$.

$\therefore 2a_n = 3a_n - 3a_{n-1}$.

故：$\frac{a_n}{a_{n-1}} = 3$

an 是以 9 为首项，公比为 3 的等比数列

$a_n = 9 \times 3^{n-1} = 3^{n+1}$.

(2) $\because \frac{b_{n+1}}{b_n} = a_n, b_1 = 3$.

$\therefore b_n = \frac{b_n}{b_{n-1}} \cdot \frac{b_{n-1}}{b_{n-2}} \cdots \frac{b_2}{b_1} \cdot b_1$.

$= a_{n-1} \cdot a_{n-2} \cdots a_1 \cdot b_1$.

$= 3^n \cdot 3^{n-1} \cdot 3^{n-2} \cdots 3^2 \cdot 3$.

$= 3^{1+2+3+\cdots+n}$.

$= 3^{\frac{n(n+1)}{2}}$.

$\log_3 b_n = \log_3 \frac{n(n+1)}{2} = \frac{n(n+1)}{2}$.

$:\frac{1}{\log_3 b_m} = \frac{2}{n(n+1)}$.

$\sum_{i=1}^{100} \frac{1}{\log_3^{b_i}}$

$= \frac{2}{1\times 2} + \frac{2}{2\times 3} + \cdots + \frac{2}{100\times 101}$.

$= 2(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{100} - \frac{1}{101})$.

$= 2(1 - \frac{1}{101})$.

$= \frac{200}{101}$.

17．解（1）由正法是理得：$\frac{\cos A}{\sin A} = \frac{\cos B + \cos C}{\sin B + \sin C}$

∴sinAcosB+sinAcosC=cosAsinB+cosAsinC

∴sinAcosB-cosAsinB=sincosA-coscsinA

∴sin(A-B)=sin(C-A)

$\therefore$ A-B=C-A

B+C=2A

又$\because$ A+B+C=π

$A=\frac{\pi}{3}$.

$(2)2b^2+c^2=(\frac{a}{\sin A})^2(2\sin^2 B+\sin^2 C)$

$=\cdot\frac{4}{3}a^2(2\sin^2 B+\sin^2 C)$.

$2\sin^2 B+\sin^2 C$.

$=2\sin^2 B+\sin2\,(\frac{2\pi}{3}-13)$.

$=2\sin^2 B+\frac{3}{4}\cos^2 B+\frac{\sqrt{3}}{2}\sin B\cos B+\frac{1}{4}\sin^2 B$

$=\frac{3}{2}\sin^2 B+\frac{\sqrt{3}}{2}\sin B\cos B+\frac{3}{4}=\frac{3}{2}\frac{1-\cos 2B}{2}+\frac{\sqrt{3}}{4}\sin 2B$

$=\frac{\sqrt{3}}{4}\sin 2B-\frac{3}{4}\cos 2B+\frac{1}{2}=\frac{\sqrt{3}}{2}\sin(2B-\frac{\pi}{3})+\frac{3}{4}\le\frac{\sqrt{3}}{2}+\frac{3}{4}$.

$\therefore 2b^2+c^2\le(\frac{2\sqrt{3}}{3}+2)a^2=6+2\sqrt{3}$.

$\therefore a^2=3$.

$\therefore a=\sqrt{3}$.

18. （1）证明:$f(-x-1)=a(-x-1)+\frac{2}{e^{-x}+1}+a-1=ax+\frac{2e^x}{e^x+1}-1$

$f(x-1)=a(x-1)+\frac{2}{e^{x+1}}+a-1=ax+\frac{2}{e^x+1}-1$.

$\therefore f(-x-1)+f(x-1)=\frac{2ex}{e^x+1}+\frac{2}{e^x+1}-ax+ax-2=0$

$\therefore$ y=f(x-1)奇函数

$(2).f'(x)=a+\frac{-2e^{x+1}}{(e^{x+1}+1)^2}$.

$=a+\frac{-2}{e^{x+1}+\frac{1}{e^{x+1}}+2}$.

$\ge a+\frac{-2}{2\sqrt{e^{x+1}\frac{1}{e^{x+1}}}+2}=a-\frac{1}{2}$

(3) $f(x) = a(x+1) + \frac{2}{e^{x+1}+1} - 1$　　显然 f(-1)=0

由(1)可得 f（x）周象关于（-1,0）对称

$f(x)$三个零点，等价于 f（x）在（-1，+∞）有 1 个零点

令$t = x+1 > 0$

$g(t) = at + \frac{2}{e^t+1} - 1 = 0$ 在(0，+∞)中的∵有 1 个零点

$\therefore a = \frac{1}{t} + \frac{-2}{t(e^t+1)} = h(t),$

$$h'(t) = -\frac{1}{t^2} + \frac{2(t+1)e^t+2}{t^2(e^t+1)^2} = \frac{-(e^t+1)^2+2(t+1)e^t+2}{t^2(e^t+1)^2} < 0$$

$\therefore h(t)$在$(0,+\infty)$单减

$\therefore a < \lim\limits_{t\to 0^+} h(t) = \frac{1}{2}$

$a \in (0, \frac{1}{2})$

19.解：（1）$\begin{cases} \log_4{}^m + 1 > 1 \\ \log_2{}^m - \log_4{}^m = \log_2{}^{\sqrt{m}} > 1 \end{cases}$

解得：$m \in (4,+\infty)$

(2) $d = a_{n+1} - a_n > 1$

$S_n = n + \frac{n(n-1)}{2}d = \frac{d}{2}n^2 + (1 - \frac{d}{2})n$

$\frac{d}{2}n^2 + (1 - \frac{d}{2})n \le n^2 + \frac{1}{2}n.$

$d \le \frac{2n-1}{n-1} = \frac{2(n-1)+1}{n-1} = 2 + \frac{1}{n-1}.$（$n \ge 2$）

∴d≤2 综上得：　1<d≤2

(3) $a_{n+1} - a_n > 1,\quad \frac{1}{3}(a_{n+1} - a_n) \le 1.$

$\therefore 1 < a_{n+1} - a_n \le 3.$